Graph reconstruction via subgraph queries

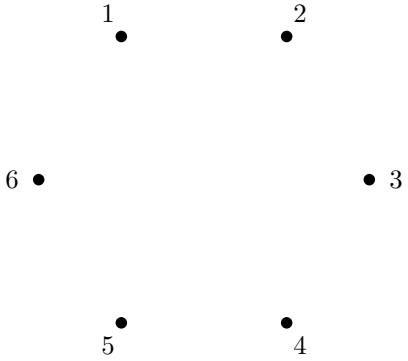
Bianca Oancea

LISN, Université Paris-Saclay

JITA, October 2026

Discovering a labelled graph

Discover which of the 15 edges are there!



$n = 6,\ k = 3$

$\binom{6}{3}$ sets of vertices we can ask about

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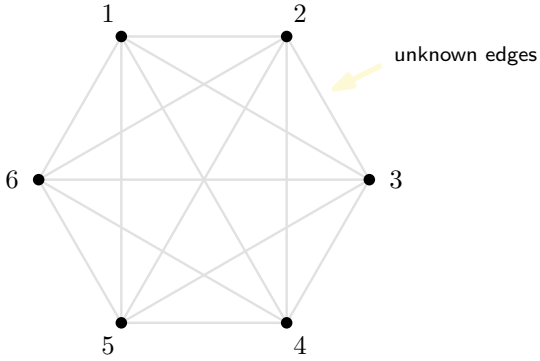

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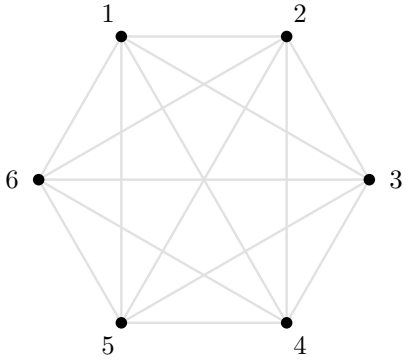

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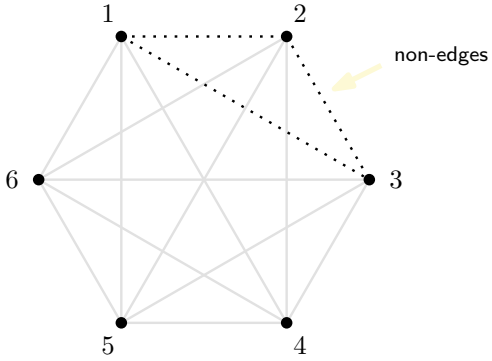

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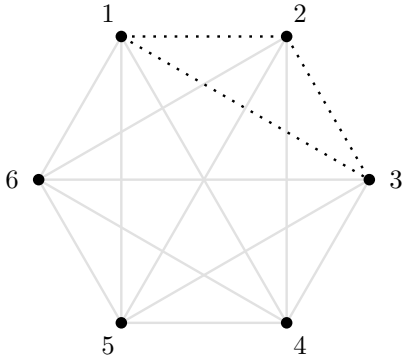

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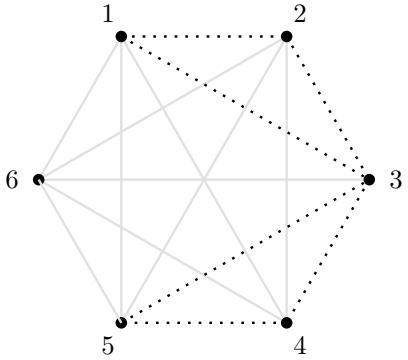

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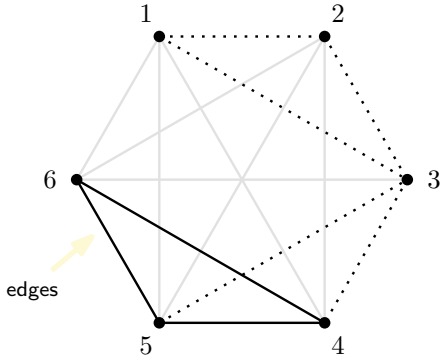

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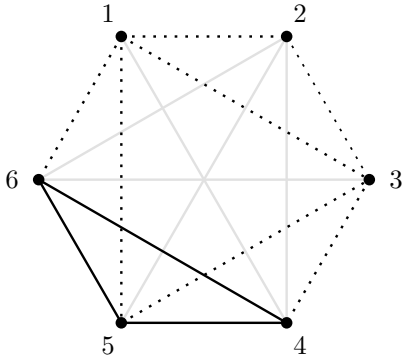

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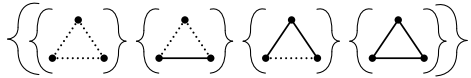
Every graph on $n \geq 3$ vertices is determined, up to isomorphism, by the collection of its unlabeled induced subgraphs on $n - 1$ vertices.



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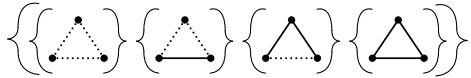

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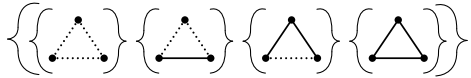

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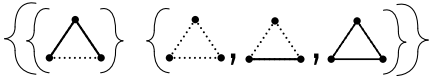
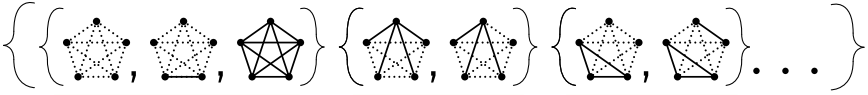
Queries on P_3- structures (Hayward, 1996)



Which partitions give enough information?

$\Pi =$ 



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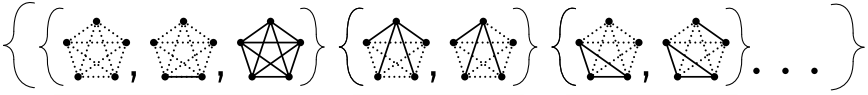
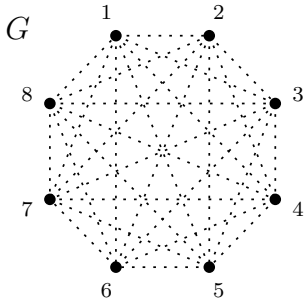
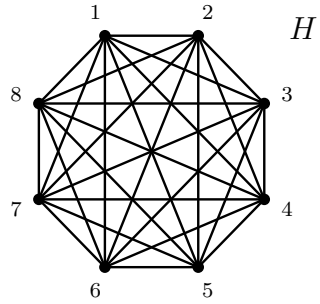
$\Pi =$ 





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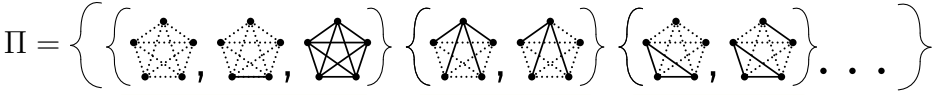
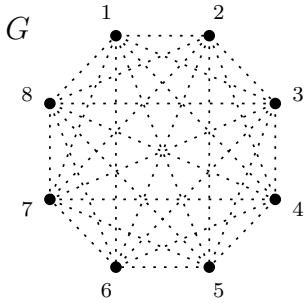
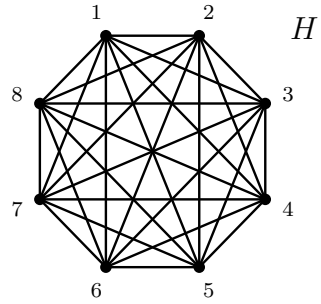
$\Pi =$ 



$\Pi(G[12345]) \;=\; A$



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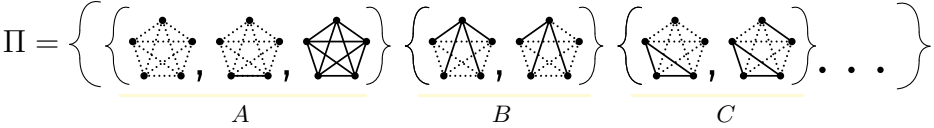

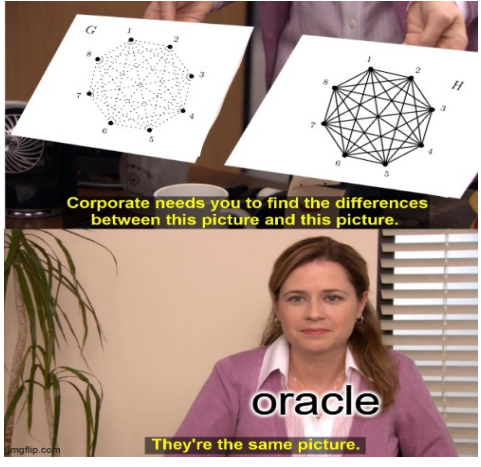

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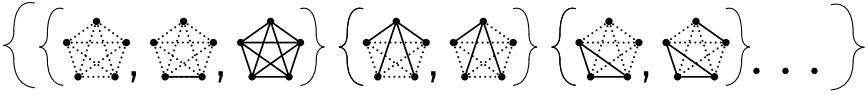
$\Pi =$ 

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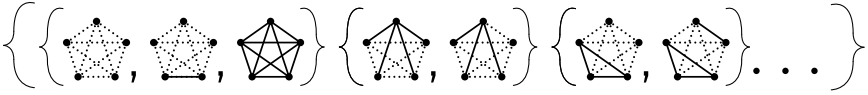
$\Pi =$ 



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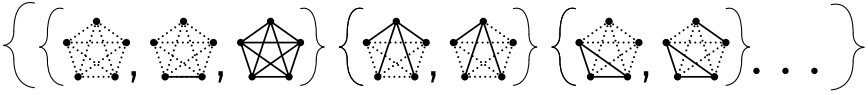
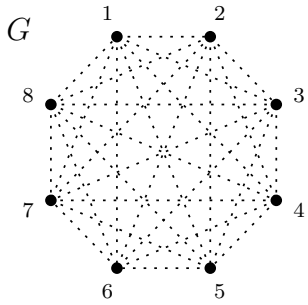
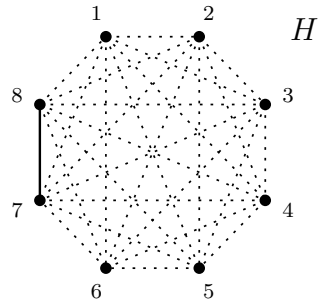
$\Pi =$ 





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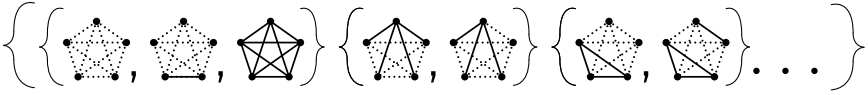
$\Pi =$ 

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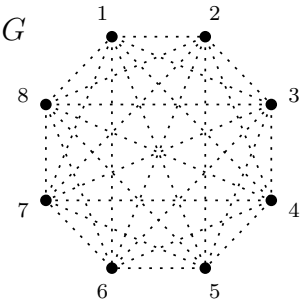

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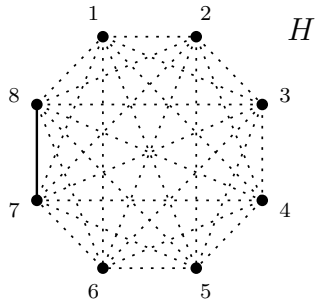

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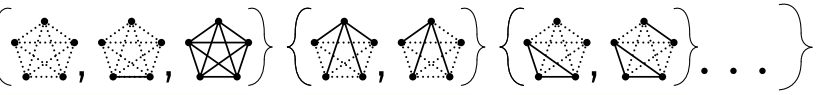
$\Pi =$ 

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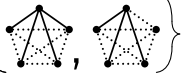
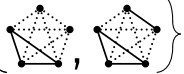
$\Pi = \Big\{ \big\{$  $\big\}$ $\big\{$  $\big\}$ $\big\{$  $\big\} \ldots \Big\}$

A B C

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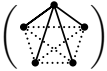
$\Pi\Big($  $\Big) \neq \Pi\Big($  $\Big)$

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$+$ complements

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Cohen, La, O., 2026+

The partition determines every graph on $n \geq N(k)$ vertices, for any $k \geq 4$, **if and only if** it satisfies these nine separations.

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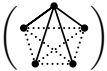
$\Pi($  $) \neq \Pi($  $)$

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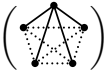
$\Pi($  $) \neq \Pi($  $)$

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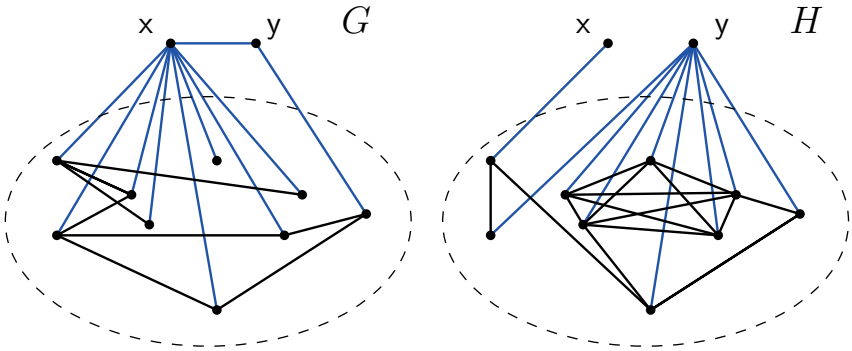
→ the nine separations correspond to $r = 2$, there is an explicit family of separations for every $r \geq 3$.

Sketch of proof



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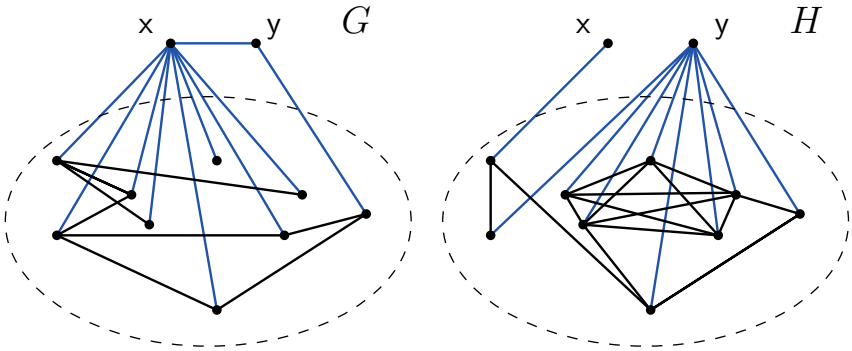
Goal: find S, $|S| = k$, with $\Pi(G[S]) \neq \Pi(H[S])$

$G \neq H$



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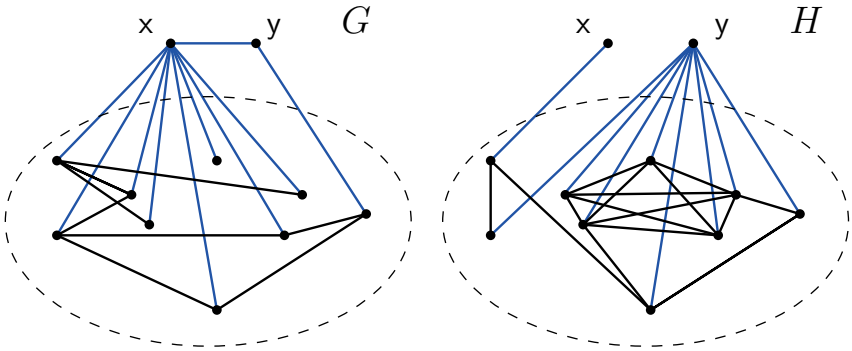

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pigeonhole principle

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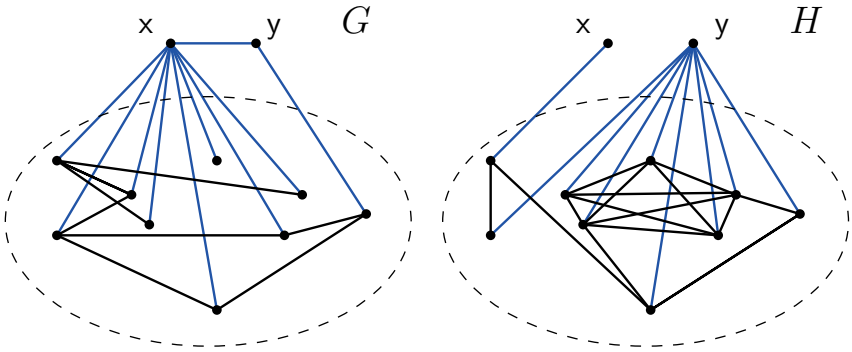

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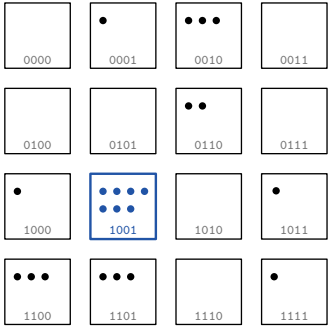

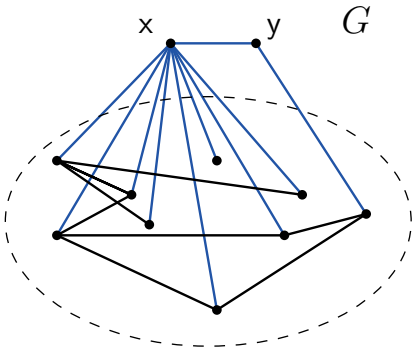

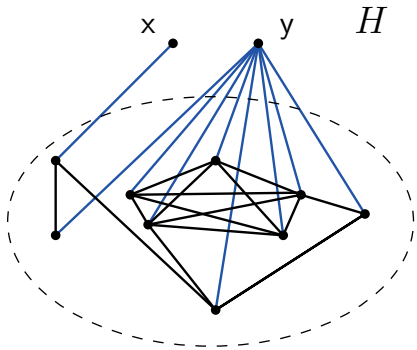

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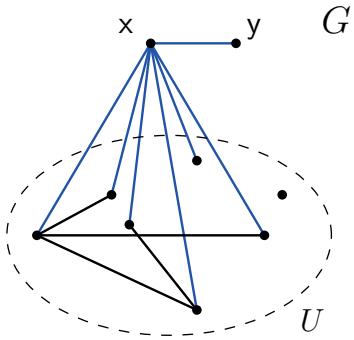

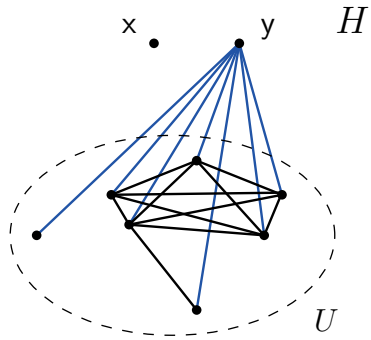

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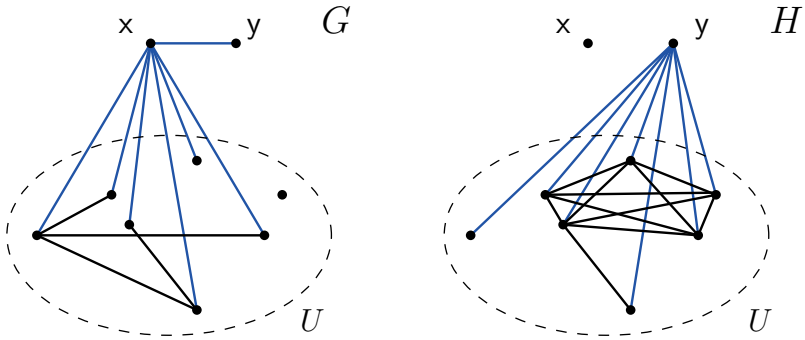

Step 2: find a subset $D \subseteq U$, $|D| \geq k$ such that D induces a complete or empty graph in each graph, G and H

Sketch of proof

Goal: find S, $|S| = k$, with $\Pi(G[S]) \neq \Pi(H[S])$

multicolor Ramsey theorem

For all integers $c \geq 1$ and $k \geq 2$, there is an integer N such that every coloring of the pairs of an N-element set with c colors has a set of size k whose pairs all have the same color.

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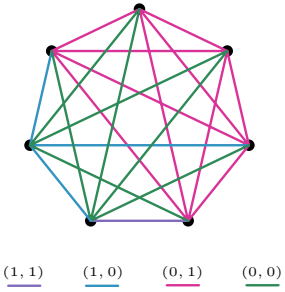
Sketch of proof

four-color Ramsey theorem



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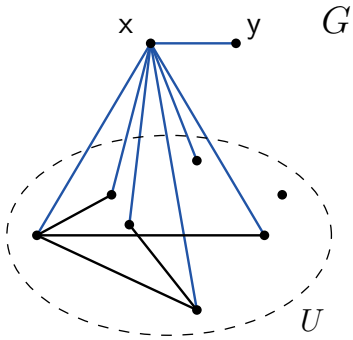

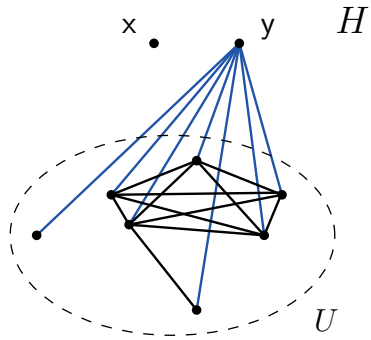

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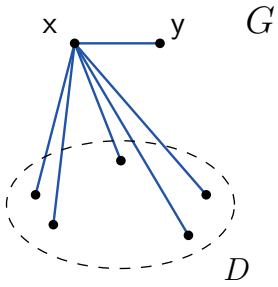

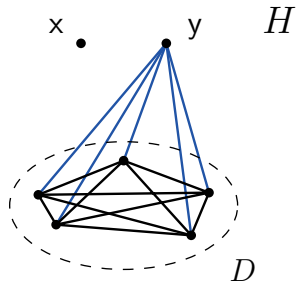

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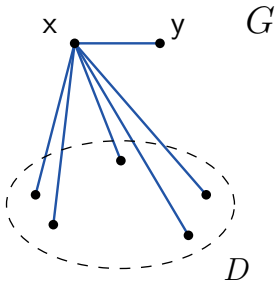

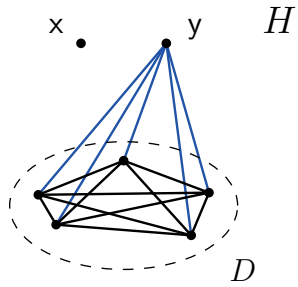

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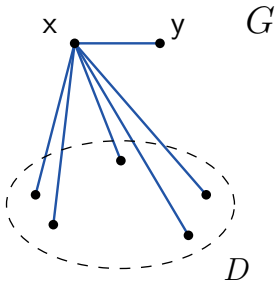

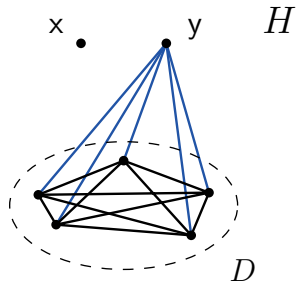

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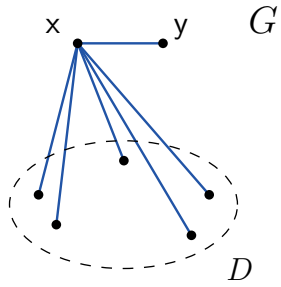
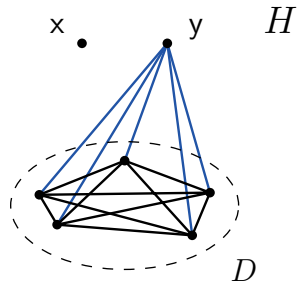


$(0, 1, 0, 1)$



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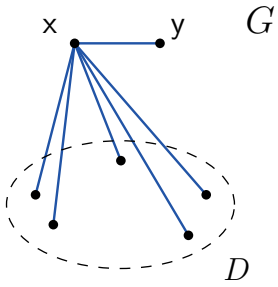

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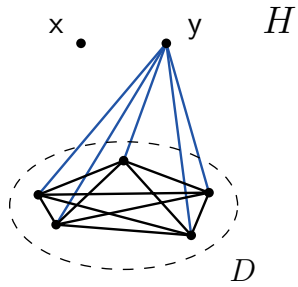

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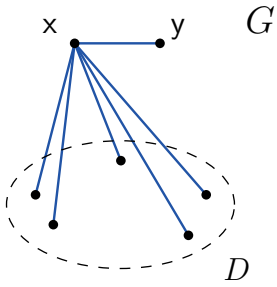
Π  $\neq$ Π 



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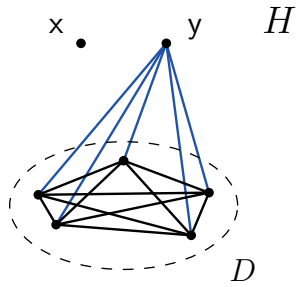

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From edge patterns to the nine separations

Pattern:
(D, xD, yD, xy)

From edge patterns to the nine separations

Pattern:
$(D,\, xD,\, yD,\, xy)$

G:
$(a,\, b,\, c,\, d)$

H:
$(a',\, b',\, c',\, d')$

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First difference

Which S to query

The induced graphs

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First difference	Which S to query	The induced graphs
$a \neq a'$	k vertices of D	
$a = a', b \neq b'$ or $a = a', c \neq c'$	x (or y) $\cup$ $k-1$ vertices of D	

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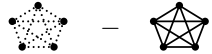
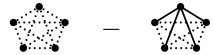
First difference	Which S to query	The induced graphs
$a \neq a'$	k vertices of D	
$a = a', b \neq b'$ or $a = a', c \neq c'$	x (or y) $\cup$ $k-1$ vertices of D	 or their complements

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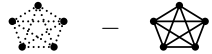
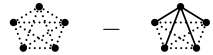
$a \neq a'$	k vertices of D	
$a = a', b \neq b'$ or $a = a', c \neq c'$	x (or y) $\cup\, k-1$ vertices of D	 or their complements
$a = a', b = b', c = c'$ $d \neq d'$	$\{x, y\} \cup k-2$ vertices of D	

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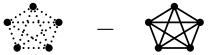
First difference	Which S to query	The induced graphs
$a \neq a'$	k vertices of D	
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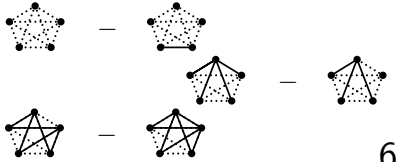
$a = a', b = b', c = c'$ $d \neq d'$	$\{x, y\} \cup k-2$ vertices of D	 or their complements

Conclusion and open questions

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Cohen, La, O., 2026+

$N(k) = 2^{\Theta(k)}$

The partition determines every graph on $n \geq N(k)$ vertices, for any $k \geq 4$, **if and only if** it satisfies these nine separations.

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Future directions

Conclusion and open questions

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Future directions

Additional conditions on the partition that lower the threshold on n

A more efficient reconstruction algorithm

Generalize the reconstruction theorem for directed graphs, edge-coloured graphs, posets, etc

Thank you

All nine separations

$\Pi($  $) \neq \Pi($  $)$

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